

CSM – 69/19
Statistics
Paper – II

Time : 3 hours

Full Marks : 300

The figures in the right-hand margin indicate marks.

Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any **three** of the remaining questions, selecting at least **one** from each Section.

SECTION – A

1. Answer any **five** of the following :

- Explain statistical basis of a control charts. If we take $N(\mu, \sigma^2)$ and replace μ and σ by their estimate \bar{x} and s then, $\bar{x} \pm \frac{3s}{\sqrt{n}}$ and if all the observations lie within the limits, the process may not necessarily in control, show it. 12
- Define survival function, cumulative distribution function, density function, Hazart

rate for exponential distribution will parameter λ , find $F(x)$: If T represent longevity then

$$E(T) = \frac{1}{\lambda} \text{ and } E(x) = \frac{1}{\lambda}, \text{ prove it.} \quad 12$$

- (c) Discuss time series and give examples of additive and multiplicative model. If $Y = Ae^{BT}$, then obtain the estimate of A and B . Indicate the difference of seasonal and cyclical components. 12
- (d) Explain the uses of index number and out of Laspeyre's, Paasche's and Fisher's ideal number which is better ? For the data : 12

Year	1965		1970	
	p_0	q_0	p_1	q_1
	(1000 Rs)	kg	(1000 Rs)	kg
	2	3	3	4
	4	2	5	3
	5	4	7	5
	3	1	4	1

Find Fisher Ideal Index Number.

- (e) Explain (i) linear model and its application,
(ii) least squares and generalised least squares method of estimation. For the data :

x	y
1	6
2	17
3	34
4	57

Obtain the equation $y = a + bx + cx^2$ using least square theory. 12

- (f) Discuss (i) Industrial production, (ii) Methods of collection of official statistics and (iii) Official agencies responsible for data collection. 12

2. (a) Discuss objective of Statistical Quality Control. Express (\bar{X}, σ) charts and R charts when standards are known and unknown. In

$N(\mu, \sigma^2)$, the unbiased estimate of σ is $\frac{s}{c}$.

Find $E(s)$.

15

- (b) Explain the sampling plan for attribute with respect to Consumer's Risk (CR) and Producer's Risk (PR) in single sampling plan. If number of defectives follows Binomial with parameter (n, p) , then find PR and CR. Whether PR and CR can be taken as Type I and Type II error in testing of hypothesis? 15
- (c) Describe AOQ and AOQL Curve's. Express goodness of a Sampling Inspection Plan with three basic criteria : OC, ASN and AOQ. 15
- (d) Discuss (i) single specification limit from $N(\mu, \sigma^2)$, when σ^2 is known. (ii) Double specification limit from $N(\mu, \sigma^2)$ when σ^2 is unknown. Whether AOQL plan is corrective sampling plan? 15
3. (a) Discuss reliability and maintainability of the system. Define hazard rate and find hazard rate of function $f(t) = \lambda e^{-\lambda t}$, $\lambda > 0$, $0 \leq t < \infty$ and show that intensity function and hazard function are same. 15

- (b) Describe exponential model and discuss I. F. R and D. F. R. model. In two parameter

exponential density $\frac{1}{\theta} e^{-\left(\frac{x-A}{\theta}\right)} x > A, \theta > 0$

find expected value of $T_r = \frac{\sum_{i=1}^r x_i + (n-r)x_r}{r}$

and $V(T_r)$. 15

- (c) Define time series and discuss trend and seasonal components with examples. Discuss the methods to determine the seasonal components. 15

- (d) Describe negative exponential distribution with censored truncated situations. In exponential distribution find the improved estimator $(C\bar{x})$ for θ and find variance for that. 15

4. (a) Discuss ARIMA model and how the orders of autoregressive and moving average components are discussed. Explain Moving average method to determine the trend. 15

- (b) Describe factor-reversal test, circular test and dimensional invariance test. Where they are used ? Compare time-reversal and factor-reversal test. 15
- (c) Describe multi-collinearity, autocorrelation, heteroscedasticity of disturbances and its testing. Please give the situations where techniques are used and what are the advantages ? 15
- (d) Describe (i) Official statistical system in India relating to population and their interpretation.
(ii) Official statistics system in India relating to agriculture and its advantages.
(iii) Reliability and limitation and publication of above official statistical system. 15

SECTION – B

5. Answer any **five** of the following :

- (a) Discuss operational research and describe simulation and Monte-Carlo methods with their structure and formulation. 12

(b) Describe (i) Simplex Method, (ii) Duality theory of Linear Programming and (iii) Transportation and assignment problem and express the economic interpretation of those methods. 12

(c) Express transition probability matrix and Poisson process. Discuss M/M/1 and M/G/1 queues. 12

(d) Explain (i) Measures of fertility, (ii) Standard death rate, (iii) Abridged life table and (iv) Logistic population growth curves. 12

(e) Write census and health surveys and use of hospital statistics. Discuss Means and show that $AM \geq GM \geq HM$. 12

(f) Give the measurements and the uses of (i) Z-scores, (ii) Standard scores and (iii) T-scores. What are the uses of factor analysis ?

6. Answer any **three** sub-parts of the following :

- (a) Discuss two-phase method. Show that the following system of linear equation has a degenerate solution : 20

$$2x_1 + x_2 - x_3 = 2$$

$$3x_1 + 2x_2 + x_3 = 3$$

- (b) Explain transportation or assignment problems. Determine an initial basic feasible solution to the following T. P. using the row minima method : 20

TO

	A	B	C	
I	50	30	220	1
From II	90	45	170	3 Availability
III	250	200	50	4
	4	2	2	
	Requirement			

- (c) Explain Poisson Process and its properties. If the time (t) to complete the service on a customer follow $s(t) = \mu e^{-\mu t}$ then show that : 20

- (i) $P[\text{no service in } \Delta t] = 1 - \mu \Delta(t) + O(\Delta t)$
 (ii) $P[\text{one service in } \Delta t] = \mu \Delta(t) + O(\Delta t)$

- (d) (i) Discuss Markov Chains and properties of finite Markov Chains. 12
- (ii) Transition probability matrix and its applications. 8

7. Answer any **three** of the following :

- (a) (i) Explain demographic data which are considered in Census 2001 and what are the limitations and uses. 12
- (ii) Discuss NSS and its uses. 8
- (b) Discuss the followings with its uses : 20
- (i) Morbidity rate
- (ii) Standardized death rate
- (iii) Infant mortality rate
- (iv) Measures of fertility
- (c) (i) Discuss life tables and its components. Express the logistic model and its fitting. 15
- (ii) Standard classification of cause of death. 5

- (d) The Markov Chain having the state space $(0, 1, 2)$ and transition matrix.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1-p & 0 & p \\ 0 & 1 & 0 \end{bmatrix}$$

Find P^2 and show that it is equal to P^4 . Also find P^n , $n > 1$. 20

8. Answer any **three** of the following :

- (a) Express life testing and how exponential model is used. In exponential model $f(x/\theta) =$

$$\frac{1}{\theta} e^{-x/\theta}, x > 0, \text{ the type II Censored data is}$$

used then prove that $\frac{2T_r}{\theta}$ follows Chi-square

distribution with $2r$ d.f. Also find the variance of (T_r) . 20

- (b) Discuss any **two** of the following : 20

- (i) Analytical structure of inventory problems
- (ii) Elements of queueing theory and G/M/1 queues

- (iii) Graphical and algebraic method of solution.
- (c) Explain factor analysis and path analysis in Psychometry. 20
- (d) Discuss percentile scores, intelligence quotient, validity of test. Explain the procedure for determination of these. 20

