| CSM – 68/19 |
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| Statistics |
| Paper – I |

*Time : 3 hours*

*Full Marks : 300*

*The figures in the right-hand margin indicate marks.*

*Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any* ***three*** *of the remaining questions, selecting at least* ***one*** *from each Section.*

## SECTION – A

1. Attempt any **three** of the following sub-parts : $20 \times 3 = 60$

(a) Let $\{X_n\}$ be a sequence of random variables. If $X_n \xrightarrow{a.s.} X$, where X is a random variable, then show that $g(X_n) \xrightarrow{a.s.} g(X)$ where g is a continuous function.

(b) Explain the properties of characteristic function. Hence using characteristic function, find out the mean and variance of Poisson distribution.

(c) Show that if $X_3 = aX_1 + bX_2$, the three partial correlations are numerically equal to unity, $r_{13.2}$ having the sign of a, $r_{23.1}$, the sign of b and $r_{12.3}$, the opposite sign of a/b.

(d) If $X \sim N(\mu, \Sigma)$, then prove that $Y = CX$ is distributed according to $N(C\mu, C\Sigma C')$ for C non singular.

2. (a) State and prove Khinchine's weak law of large numbers.

(b) X and Y are two random variables having the joint density function $f(x, y) = \frac{1}{27}(2x + y)$, where x and y can assume only the integer values 0, 1 and 2. Find the conditional distribution of Y for X = x.

(c) A coin is tossed until a head appears. What is the expectation of the number of tosses required ? $20 \times 3 = 60$

3. (a) Show that Poisson distribution is a limiting case of the negative Binomial distribution.

(b) If X and Y are independent Gamma variates with parameters $\mu$ and $\upsilon$ respectively, show that $u = X + Y$, $Z = \frac{X}{Y}$ are independent and that u is a $\Gamma(\mu+\upsilon)$ variate and Z is a $\beta_2(\mu, \upsilon)$ variate.

(c) State and prove Gauss-Markov theorem and explain its applications in linear estimation.

$20 \times 3 = 60$

4. (a) If $r_{12}$ and $r_{13}$ are given, show that $r_{23}$ must lie in the range :

$$r_{12}r_{13} \pm \left(1 - r_{12}^2 - r_{13}^2 + r_{12}^2\, r_{13}^2\right)^{1/2}$$

If $r_{12} = k$ and $r_{13} = -k$, then show that $r_{23}$ will lie between $-1$ and $1 - 2k^2$.

(b) What is the relation between Hotelling's $T^2$ statistic and Mahalanobis $D^2$ statistic. Also show that $T^2$ is invariant under any linear transformation.

(c) Let $X_1, X_2, \cdots\cdots X_N$ be N independent observation vectors distributed according to

$N_P(\mu, \Sigma)$. Then show that the marginal distribution of any subset of p-vectors is also a multivariate normal. 20×3 = 60

## SECTION – B

5. Answer any **three** of the following sub-parts :

20×3 = 60

(a) Show that the maximum likelihood estimators are consistent.

(b) State and prove Wald's fundamental identity.

(c) Prove that in simple random sampling without replacement, the sample mean square is an unbiased estimate of the population mean square.

(d) Derive expressions to compare the efficiencies of L. S. D. with respect to R. B. D. and C. R. D.

6. (a) Show that an M. V. U. estimator is unique in the sense that if $T_1$ and $T_2$ are M. V. U. estimators for $y(\theta)$, then $T_1 = T_2$ almost surely.

(b) State and prove Cramer-Rao inequality.

(c) Find out the B. C. R. for $H_0 : \sigma = \sigma_0$ against the alternative $H_1 : \sigma = \sigma_1$ for the normal distribution with zero mean and variance $\sigma^2$.

20×3 = 60

7. (a) What are the advantages and disadvantages of non-parametric method over parametric methods ? Discuss the Mann-Whitney-Wilcoxon test for the equality of two population distribution functions.

(b) Explain how SPRT differs from the Neyman-Pearson test procedure. Derive the OC function and ASN for the SPRT.

(c) What is the regression method of estimation ? Compare the precision of the regression estimate with that of the ratio estimate.

8. (a) Explain systematic sampling and discuss its advantages and disadvantages. Also obtain

an estimate of the population variance based on the above method.

(b) Describe the layout of a $2^3$ experiment where all the interactions are partially confounded. In such a case indicate d. f. s. and s. s's for all the components of treatment sum of squares.

(c) Define a BIBD. State the important relations among the parameters of a BIBD and prove any two of them. 20×3 = 60